Technical Comments

Comment on "Exact Method of Designing Airfoils with Given Velocity Distribution in Incompressible Flow"

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STRAND'S paper[1] on the modification of Arlinger's method of airfoil design[2] contains some misleading statements and raises some questions regarding the improvements the new method offers. The remark in the introduction that "no really satisfactory exact inverse method yet exists for computing profile shapes from a specified velocity distribution," and the contention that specification of the velocity distribution in the circle plane is "of very limited usefulness" are misleading justifications for Strand's work. The problem of determining the airfoil shape which corresponds to a velocity distribution specified as a function of arc length was solved by Mangler[3] in 1938. Since then several other exact airfoil design methods have been developed (e.g., Lighthill,[4] Peebles,[5] and Eppler[6]). Peebles' method was used at Douglas for many years, and several hundred airfoils were designed using a desk calculator. Eppler's method, which requires that the velocity distribution be specified in the circle plane, has been used to design the airfoils which are used on several high-performance sailplanes. While other examples could be cited, the main point to consider is that an arbitrarily specified velocity distribution will not necessarily provide a corresponding closed and nonreentrant airfoil, and consequently some "man-in-the-loop" interaction is inevitable. The utility of any airfoil design method is therefore somewhat dependent on the intuition and ingenuity of the user.

Strand claims to have improved the utility of Arlinger's work on the basis that the method of Ref. 2 causes "large undesirable waviness in the modified velocity distribution along the lower surface" for cases where the input velocity distribution is not close to the output or "accepted" velocity distribution. However, the example airfoil design given in Ref. 1 also exhibits significant waviness in its lower surface output velocity distribution. In addition, there is a rather large difference between the input and output velocity distributions on the lower surface near the trailing edge.

It would, therefore, be reassuring if Strand would provide some additional examples to demonstrate the validity of his modification. These should include a comparison of the results obtained using Arlinger's method with and without Strand's modification for the same input velocity distribution. This would permit an evaluation of the reduction in the output velocity distribution waviness together with any other improvements or deficiencies which are a consequence of the modification. Another logical test case is a Joukowski or Karman-Trefftz airfoil where the analytic solution is known and a direct comparison with the calculated result can be made. In this case, particular attention should be paid to the details of the flow in the leading-edge stagnation point and trailing-edge regions.

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Reply by Author to R. H. Liebeck

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I BELIEVE that my statements, as quoted above by R.H. Liebeck, correctly characterize the state of the art with respect to exact inverse methods of airfoil design. The methods of Mangler, Lighthill, Peebles, and Eppler are all closely related. The airfoil shape is found from a velocity distribution prescribed as a function of the angular coordinate around the circle into which the unknown shape is to be mapped. The "usefulness" of these approaches is possibly best indicated by quoting Peebles: "The closure integrals are calculated and one begins the unhappy business of surrendering a bit here and a bit there of the aerodynamically good qualities until finally closure is reached, each change representing a new calculation to determine the effect on closure. When the best compromise between closure and ideal characteristics has been reached, $\tau^*(\varphi)$ must be calculated. . . ." Arlinger obviously determined that these methods should be improved, and recently developed an approach by which the given velocity distribution could be prescribed naturally as a function of the distance along the airfoil surface and by which all compromises are made along the lower surface. It is noted that Liebeck himself did not employ any of the available exact inverse methods to calculate the shapes of his high-lift airfoils in Ref. 1, but rather employed second-order linear theory. In his next paper[2] he used a combination inverse-direct exact method which is quite dependent on the intuition and ingenuity of the user.

As mentioned in my paper,[3] the small-scale waviness in the modified velocity distribution along the lower surface results from the use of a large but finite, rather than infinite, number of terms in my sine series expansion of this function. In Fig. 1 (equivalent to Fig. 2 of Ref. 3) the lower surface velocity distributions for 30 and 3 terms are compared. The 3-term curve was obtained by running my digital computer program with $\lambda_4 = \lambda_5 = 0$. It is clear that the waviness is magnified by using only three terms and no minimization of the difference between the modified and the prescribed distribution.

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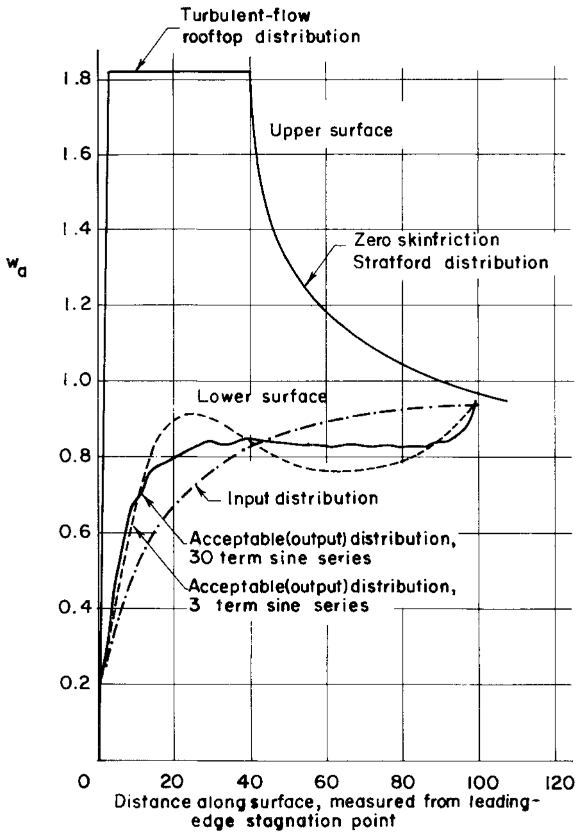

Fig. 1 Velocity distribution for a sample case, $\Gamma = 68$. Stratford distribution calculated for $Re = 3 \times 10^6$.

Arlinger utilized a slightly different orthogonal 3-term series expansion than that used here. He also let $\theta_2 = \pi + 2\alpha$, while in my case θ_2 is slightly larger than $\pi + 2\alpha$. Unfortunately my Arlinger computer program deck is no longer intact. Hence a direct comparison with the Arlinger approach is not possible without some expense. The 3-term curve in Fig. 1 is, however, very representative of many similar curves obtained earlier. I might also mention that the small-scale waviness of the 30-term series does not show up as as waviness in the airfoil contours because the wavy curve was smoothed before the airfoil coordinates were calculated.

The rather large difference between the input and output velocity distributions on the lower surface near the trailing edge is, of course, caused by my particular choice of minimizing function, i.e.,

$$H(C_k) = \int_{\theta_2}^{2\pi} [\ln \frac{w_a}{w_{a_o}}]^2 d\theta =$$

$$\int_{\theta_2}^{2\pi} [\frac{w_a - w_{a_o}}{w_{a_o}} + \text{higher order terms}]^2 d\theta \qquad (1)$$

This function is obviously heavily weighted in favor of the distances near the leading and trailing edges. It would be a definite improvement if a different minimizing function could be found which would be more linear with distance along the surface in the physical plane.

The minimization procedure outlined in Ref. 3, in view of Eq. (1), should be expected to work only when

$$[w_a(\theta) - w_{a_o} \ll (\theta)]/w_{a_o}(\theta) \ll 1$$

all along the correction interval, cause only in this case is the difference between the modified and the prescribed velocity distribution minimized. Thus the function M (or H) of Ref. 3 must always be chosen close to its minimum value (see Fig. 5 of Ref. 3).

The correctness of the inverse method developed was tested with a Joukowsky symmetrical airfoil at angle of attack. The result of this test was not reported in the paper since it only verified that the digital program worked satisfactorily, namely that with a hand-calculated Joukowsky velocity distribution as input, the digital program gave the identical velocity distribution as output, and that the corresponding output airfoil shape was the symmetrical shape from which the velocity distribution had been hand calculated. Karman-Trefftz airfoils can not, of course, be used as a comparison, since they have nonzero trailing-edge angles, requiring a stagnation point at the trailing edge. A further modification of the present digital program would be required to handle this option.

References

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[3] Strand, T., "Exact Method of Designing Airfoils with Given Velocity Distribution in Incompressible Flow," *Journal of Aircraft*, Vol. 10, Nov. 1973, pp. 651-659.